Resistors connected in parallel

The aim of this article is to consider how voltage, current and power are affected by parallel connected loads. This will be complemented using a few examples.

Introduction

It should be remembered that the resistance of a material is dependent upon the:

- conductor material (resistivity),
- length,
- cross-sectional area, and
- ambient temperature.

It is important that the correct methodology is followed when dealing with such connected loads. Parallel or shunt connected loads are connected 'across' the supply, whilst series connected loads are connected 'in line' or series with the supply. Only purely resistive loads will be considered in this article.

Note: A shunt resistor typically provides an alternative low resistance path for the flow of current in a circuit.

Parallel connected resistors

Resistors are connected in parallel when both of their terminals are respectively connected to each terminal of the other resistor(s) as shown in Fig 1.

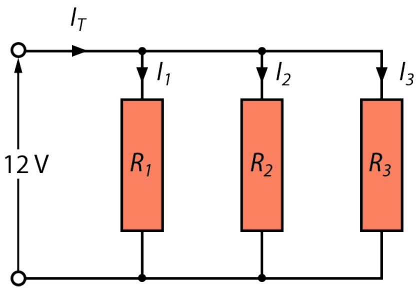
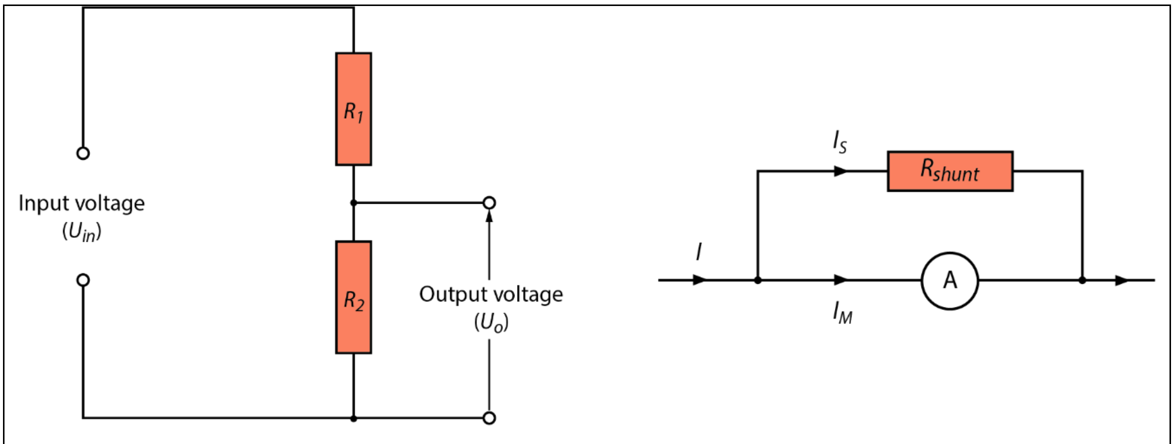


Fig 1. Resistors connected in parallel

Unlike a series circuit, the circuit current in a parallel network can take more than one path.
In which case, the current may not be the same through all the branches in the parallel network, and therefore can be classified as being a current divider – much like a series circuit could be described as being a voltage divider. Fig 2, shows the comparison between the two types of circuits.



a) Series. Voltage divides across R_1 and R_2

b) Parallel. Current divides between the shunt resistor and the current through the ammeter

Fig 2. Comparison of a series and parallel circuit

Although parallel and series circuits differ from one another, the equations of Ohm's Law equally apply to both types of circuit.

The total resistance for a parallel circuit cannot be found by adding together the resistances, as is the case for a series circuit; a different approach is required.

Considering Fig 1 in terms of current flow:

$$I_{\text{Total}} = I_1 + I_2 + I_3$$

From Ohm's Law, the current drawn from the supply is:

$$I = \frac{U}{R_t}$$

Substituting I for $\frac{U}{R}$ wherever it occurs gives:

$$\frac{U}{R_T} = \frac{U}{R_1} + \frac{U}{R_2} + \frac{U}{R_3}$$

Cancelling out U because the voltage is constant:

$$\frac{\cancel{U}^1}{R_T} = \frac{\cancel{U}^1}{R_1} + \frac{\cancel{U}^1}{R_2} + \frac{\cancel{U}^1}{R_3} \text{ gives } \frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} \ldots + \frac{1}{R_n}$$

Normally, when calculating total resistance of resistors in parallel, it is easiest to use the reciprocal key on your calculator, which is typically denoted as $\frac{1}{x}$ or x^{-1}.

Remember, the reciprocal key must be pressed after the = key in order to get the correct answer. Another way of determining the total resistance of a parallel circuit would be to use the product over the sum rule:

$$R_T = \frac{R_1 \times R_2}{R_1 + R_2}$$

However, this formula only works best when there are two resistors in parallel. Where there are more resistors connected in parallel a variant on this formula can be used, but it becomes unwieldy.

Note: The combined resistance for a parallel circuit is always less than the lowest value connected, as shown in example 1.

Example 1

Resistors of 3 Ω, 5 Ω and 8 Ω are connected in parallel. What is their combined resistance?

$$\frac{1}{R_T}=\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}=\frac{1}{3}+\frac{1}{5}+\frac{1}{8}$$

$$\frac{1}{R_T}=0.3333+0.2+0.1250=0.6583$$

$$R_T=\frac{1}{0.6583}=1.52\Omega$$

In a parallel circuit, since the voltage is constant, the current through each branch is dependent upon the value of resistance in that branch and can therefore be found using the current divider rule:

$$I_1=I\times\frac{R_2}{R_1+R_2}$$

Example 2

Consider the circuit in Fig 3, which shows two loads connected in parallel. Determine the current in each branch, the source voltage and the power consumed or absorbed by each load?

Note: Since power consumed by each load *must* equal the total power applied by the source(s) (as per the Law of Conservation of Energy), the manner in which the circuit is configured will have no effect on the solution process. The power formulas shown, are used whether the circuit(s) are series or parallel connected.

The power consumed by each resistor in a parallel network can be found by one of three methods:

- Method 1, using the branch current and the resistance: $P_1=I_1^2\times R_1$
- Method 2, using the branch current and the voltage dropped across a resistor: $P_1=U\times I_1$
- Method 3, using the voltage dropped across a resistor and the resistance: $P_1=\frac{U^2}{R_1}$

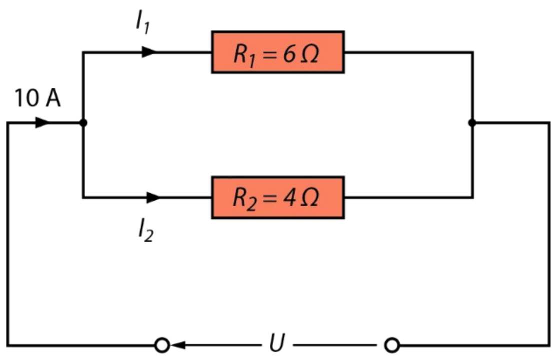


Fig 3. Parallel circuit for example 2

First, determine the branch currents by using the current divider rule:

$$I_1 = I \times \frac{R_2}{R_1 + R_2} = 10 \times \frac{4}{4+6} = 4\,A$$

$$I_2 = I \times \frac{R_1}{R_1 + R_2} = 10 \times \frac{6}{4+6} = 6\,A$$

Source voltage:

$U = I \times R$, and because it is a parallel circuit having a constant voltage either branch current and its resistance can be used.

$$U = I_1 \times R_1 = 4 \times 6 = 24\,V$$

Power consumed by each branch load:

$$P_1 = I_1^2 \times R_1 = 4^2 \times 6 = 96\,W$$

$$P_2 = I_2^2 \times R_1 = 6^2 \times 4 = 144\,W$$

This can be verified by checking against the power developed at the source:

$$P = U \times I = 24 \times 10 = 240\,W \quad (96 + 144)$$

Summary

This article has considered resistive loads connected in parallel and has shown that the current through each branch of a parallel network is dependent upon the value of resistance in that branch.